2.1 Product and Quotient Rules: Quiz C Key

Tutor copy. Every answer was checked by symbolic computation (SymPy, `verify_quiz.py`). All problems are original; provenance is in `PROVENANCE_QC.md`. Prepared with assistance from Claude, an AI model made by Anthropic.

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General grading rules

- **Show work.** A bare correct answer earns at most half credit, except where a row says otherwise.
- **Any valid method.** Simplifying first, the product rule, and the quotient rule are all accepted wherever they apply; no method is required, and long but correct work gets full credit.
- **Value vs. derivative.** A function value where a derivative was asked (or the reverse) earns 0 for that part unless a row says otherwise.
- **Domains.** On Q1, a correct note on where f' does not exist is accepted, never required.
- **Simplifying.** Answers need not be simplified beyond what a row says; equivalent forms are correct.

Quiz C

Item	Answer	Rubric / trap
1(a)	$2x\tan x + x^2\sec^2 x$	product rule: $(2x)(\tan x) + (x^2)(\sec^2 x)$. 1 pt rule applied or simplification, 1 pt correct derivative. *Trap:* multiplying the derivatives gives $2x\sec^2 x$; earns 0.
1(b)	$\dfrac{13}{(2x+3)^2}$	quotient rule: $\dfrac{1\cdot(2x+3)-(x-5)(2)}{(2x+3)^2}$. 1 pt rule applied or simplification, 1 pt correct derivative. *Trap:* $-(x-5)(2) = -2x+10$; reversing the numerator gives -13.
1(c)	0 (for every x in the domain of f)	$\dfrac{x(x^2+2)}{4x(x^2+2)} = \frac{1}{4}$ wherever f is defined, a constant. The quotient rule, carried through, also gives 0. 1 pt rule applied or simplification, 1 pt correct derivative. *Trap:* a constant in disguise. The quotient rule is valid but long; its numerator simplifies to 0.
1(d)	$-2\csc^2 x$	$\cos x \sec x = 1$ wherever $\sec x$ is defined, so $f(x) = 2\cot x + 1$. The product rule on $\cos x \sec x$ also gives 0: $-\sin x \sec x + \cos x \sec x \tan x = -\tan x + \tan x$. 1 pt rule applied or simplification, 1 pt correct derivative. *Trap:* a constant hidden in a product; the derivative of $\cot x$ is $-\csc^2 x$, with a minus sign.
1(e)	0	π^2 and $\tan\left(\frac{\pi}{4}\right) = 1$ are numbers, so $f(x) = \pi^2$ for every x. 1 pt rule applied or simplification, 1 pt correct derivative. *Trap:* no x appears; 2π (treating π as the variable) or any quotient-rule result earns 0.
2	$y + (\pi+1) = -(x-\pi)$, i.e. $y = -x - 1$	1 pt $f(\pi) = (\pi+1)\cos\pi = -(\pi+1)$; 2 pts $f'(x) = \cos x - (x+1)\sin x$; 1 pt $f'(\pi) = -1 - 0$; 1 pt equation, any correct form. *Trap:* $\cos\pi = -1$, not 1; $\sin\pi = 0$ removes the second term.
3	$x = -\frac{\pi}{4}$ and $x = \frac{\pi}{4}$	1 pt $f'(x) = 2 - \sec^2 x$; 2 pts $\cos^2 x = \frac{1}{2}$, $\cos x = \pm\frac{\sqrt{2}}{2}$; 2 pts both solutions in the interval and nothing else. *Trap:* the interval contains negative x: $\cos\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ too, so stopping at $\frac{\pi}{4}$ costs 2. $\frac{3\pi}{4}$ and $\frac{5\pi}{4}$ are outside the interval.

Item	Answer	Rubric / trap
4(a)	-5	Readings: $f(3)=1$, $f'(3)=2$, $g(3)=-2$, $g'(3)=-1$ (slopes are rise over run of the segment containing $x=3$). $p'(3)=f'(3)g(3)+f(3)g'(3)$. 2 pts readings, 1 pt rule, 1 pt value. *Trap:* $f'(3)g'(3)=-2$; $f(3)g(3)=-2$; reading a height where a slope is needed; mixing a value from one graph with a slope from the other.
4(b)	5	Readings: $f(-1)=1$, $f'(-1)=-2$, $g(-1)=2$, $g'(-1)=1$ (slopes are rise over run of the segment containing $x=-1$). $q'(-1)=\dfrac{g'(-1)f(-1)-g(-1)f'(-1)}{[f(-1)]^2}$. 2 pts readings, 1 pt rule, 1 pt value. *Trap:* q has g on top: the f/g version gives $-\frac{5}{4}$; $g'(-1)/f'(-1)=-\frac{1}{2}$; reversed numerator gives -5.
5(a)	3	$q'(x)=\dfrac{f'(x)g(x)-f(x)g'(x)}{[g(x)]^2}$, $-2=\dfrac{(2)(2)-4g'(1)}{4}$; substitute the given values and solve for $g'(1)$. 2 pts rule written with letters, 1 pt substitution, 1 pt value. *Trap:* the item gives the derivative of the quotient and asks for a factor's derivative; reversing the numerator, or $q'=f'/g'$, gives -1.
5(b)	8	$h'(x)=1\cdot f(x)+(x+1)f'(x)$, $5=f(2)+3(-1)$; substitute the given values and solve for $f(2)$. 2 pts rule written with letters, 1 pt substitution, 1 pt value. *Trap:* this part asks for a function value, $f(2)$; dropping the $1\cdot f(x)$ term gives $-3=5$, a contradiction, not an answer.
6(a)	14	$h'(2)=f'(2)g(2)+f(2)g'(2)$, with the row $x=2$. 1 pt rule, 1 pt entries, 1 pt value. *Trap:* $f'(2)g'(2)=-8$; $f(2)g(2)=-3$. Columns run x, $g'(x)$, $f'(x)$, $f(x)$, $g(x)$.
6(b)	0	$k(x)=f'(1)\,g'(3)=-9$ for every x, a constant. 1 pt, all or nothing. *Trap:* $f'(1)$ and $g'(3)$ are numbers read from the table, so k is a constant; running the product rule earns 0.

Points: $10+5+5+8+8+4=40$.